1) Done in class

2) a) Let \( x = y \implies \dot{y} = 3y - 2x \).

b) \( (x^*, y^*) = (0, 0) \).
\[
J(x^*, y^*) = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}
\]
\[
\begin{vmatrix} -\lambda & 1 \\ -2 & 3-\lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2 = 0 \implies \lambda = \frac{3 \pm \sqrt{9 - 8}}{2} = 1, 2
\]

c) \[
\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix} v_1 = 0 \implies v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} -2 & 1 \\ -2 & 1 \end{pmatrix} v_2 = 0 \implies v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.
\]

3) \[
\begin{align*}
\dot{x} &= 2x - x^2 - y \\
\dot{y} &= x - y - z \\
\dot{z} &= y - 2z
\end{align*}
\]
\[
(x^*_1, y^*_1, z^*_1) = (0, 0, 0), \quad \left(\frac{4}{3}, \frac{8}{9}, \frac{4}{9}\right)
\]
\[
y^*_2 = 2z^*_1 \implies x^*_2 = 3z^*_1 \implies 6z^*_1 - 9z^*_2 - 2z^*_1 = 0
\]
\[
\implies Z^*_1 (4 - 9z^*_1) = 0 \implies z^*_1 = 0, \frac{4}{9}
\]
\[
\implies y^*_2 = 0, \frac{8}{9}, x^*_2 = 0, \frac{4}{3}
\]

Can you find the fixed points? Yes.

What conclusions can you make? Not much who more analysis.

4) \[
\begin{align*}
\dot{x} &= 2x - x^2 - y - 2z \\
\dot{y} &= x - y - 2z \\
\dot{z} &= y - 2z
\end{align*}
\]

What can we say about this system? It's certainly more interesting than the previous one.

Can you make a hypothesis before doing any analysis? I think everything will die off; i.e., \((0, 0, 0)\) is a stable f.p. Did anyone try this out on the plane?