(1) Evaluate the integral \( \iint xy \, dA \), over the region enclosed in the first quadrant, outside the circle \( r = 1 \) and inside the circle \( r = 2 \cos \theta \).

Solution:

\[
\int_0^{\pi/3} \int_1^{2 \cos \theta} (r \cos \theta)(r \sin \theta) \, r \, dr \, d\theta = \int_0^{\pi/3} \int_1^{2 \cos \theta} r^3 \cos \theta \sin \theta \, dr \, d\theta = \int_0^{\pi/3} \frac{1}{4} r^4 \mid_1^{2 \cos \theta} \cos \theta \sin \theta \, d\theta \\
= \int_0^{\pi/3} \frac{1}{4} [16 \cos^4 \theta - 1] \cos \theta \sin \theta \, d\theta = -\int_1^{\pi/2} \frac{1}{4} [16u^4 - 1] \, du \\
= -\frac{1}{8} u^2 + \frac{2}{3} u^{1/2} \bigg|_0^{1/2} = -\frac{1}{48}.
\]

(2) Compute \( \iint_R (2x - 3) \, dA \) where \( R \) is the region enclosed by the curves \( y = x + 4 \) and \( y = x^2 - 2x \).

Solution: Intersection: \( x = -1 \) and \( x = 4 \), then

\[
\int_{-1}^{4} \int_{x^2 - 2x}^{x+4} (2x - 3) \, dy \, dx = \int_{-1}^{4} (2x - 3)(4 + 3x - x^2) \, dx = \int_{-1}^{4} (-2x^3 + 9x^2 - x - 12) \, dx \\
= \left[ -\frac{1}{2} x^4 + 3x^3 + \frac{1}{2} x^2 - 12x \right]_{-1}^{4} = 15.
\]

(3) Integrate

\[
\int_{-1}^{2} \int_{-3}^{3} (2x^2y - 3x) \, dy \, dx.
\]

Solution:

\[
\int_{-1}^{2} \int_{-3}^{3} (2x^2y - 3x) \, dy \, dx = \int_{-1}^{3} \left[ x^2y^2 - 3xy \right]_{-3}^{3} \, dx = \int_{-1}^{3} \left[ 27x^2 - 18x - 9x^2 + 9 \right] \, dx \\
= \int_{-1}^{3} \left[ 18x^2 - 9x \right] \, dx = 9x^3 - \frac{9}{2} x^2 \bigg|_{-1}^{3} = 135/2.
\]

(4) Reverse the order of integration to evaluate

\[
\int_{0}^{1} \int_{3y}^{x/3} e^{x^2} \, dx \, dy.
\]

Solution: We first reverse the domain

\[
D = \{(x, y) | 0 \leq y \leq 1, \, 3y \leq x \leq 3\} = \{(x, y) | 0 \leq x \leq 3, \, 0 \leq y \leq \frac{x}{3}\}
\]

Then

\[
\int_{0}^{3} \int_{0}^{x/3} e^{x^2} \, dy \, dx = \int_{0}^{3} \left[ e^{x^2} \right]_{0}^{x/3} \, dx = \frac{1}{3} \int_{0}^{3} x e^{x^2} \, dx = \frac{1}{6} e^{x^2} \bigg|_{0}^{3} = \frac{1}{6} (e^9 - 1).
\]
(5) Using cylindrical coordinate find the volume of the region between the paraboloid \( z = 9 - x^2 - y^2 \), the plane \( z = 0 \), and the cylinder \( x^2 + y^2 = 1 \).

**Solution:**

\[
\int_0^{2\pi} \int_0^1 \int_0^{9-r^2} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 (9-r^2) r \, dr \, d\theta = \int_0^{2\pi} \int_0^1 (9r-r^3) \, dr \, d\theta
\]

\[
= \int_0^{2\pi} \left[ \frac{9}{2} r^2 - \frac{r^4}{4} \right]_0^1 \, d\theta = \int_0^{2\pi} \frac{19}{4} \, d\theta = \frac{19}{2}\pi.
\]

(6) Use cylindrical or polar coordinates to find the volume of the region bounded by \( z = 2 - x^2 - y^2 \) and \( z = \sqrt{x^2 + y^2} \).

**Solution:** First we figure out where they intersect in cylindrical coordinates: \( 2 - r^2 = r \Rightarrow r = 1 \).

Notice that the intersection has no \( \theta \) dependence. Also, \( z = 2 - r^2 \) is on top and \( z = r \) is on the bottom

\[
\int_0^{2\pi} \int_0^1 (-r^2 - r + 2) r \, dr \, d\theta = 2\pi \left[ -\frac{1}{4} r^3 - \frac{1}{3} r^2 + r \right]_0^1 = \frac{5}{6}\pi.
\]

(7) Find the area in the \( xy \)-plane bounded by \( y = 0 \), \( x = 0 \), \( y = 1 \), and \( y = \ln x \).

**Solution:** Notice that there is only one boundary in \( x \) but three in \( y \), however \( y = \ln x \Rightarrow x = e^y \), so this must be the other boundary in \( x \).

\[
\int_0^1 \int_0^{e^y} dxdy = \int_0^1 e^y dy = e - 1.
\]

(8) Reverse the order and evaluate

\[
\int_0^\pi \int_x^\pi \frac{\sin y}{y} \, dy \, dx
\]

**Solution:** Again, lets reverse the domain first

\[
D = \{(x,y) | 0 \leq x \leq \pi, x \leq y \leq \pi \} = \{(x,y) | 0 \leq y \leq \pi, 0 \leq x \leq y \}
\]

then

\[
\int_0^\pi \int_0^y \sin y \, dx \, dy = \int_0^\pi \sin y \, dy = 2.
\]
(9) Use a triple integral to find the volume of the solid in the first octant that is bounded by \( x = 0, y = 0, z = 0, \) and 
\[
\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.
\]

**Solution:** Let's use \( x \) as our independent variable and go from there. If \( y \) and \( z \) are 0, then 
\( 0 \leq x \leq a \), next if \( z = 0 \), \( 0 \leq y \leq b - \frac{b}{a}x \), and finally \( 0 \leq z \leq c - \frac{b}{a}x - \frac{c}{b}y \). The integral becomes
\[
\int_0^a \int_0^{b-bx/a} \int_0^{c-cx/a-cy/b} dz dy dx = \int_0^a \int_0^{b-bx/a} \left( c - \frac{c}{a}x - \frac{c}{b}y \right) dy dx = \int_0^a \left[ cy - \frac{c}{a} xy - \frac{c}{2b} y^2 \right]_0^{b-bx/a} dx
\]

\[
= \int_0^a \left[ c \left( b - \frac{b}{a}x \right) - \frac{c}{a} \left( bx - \frac{b}{a} x^2 \right) - \frac{c}{2b} \left( b - \frac{b}{a} x \right)^2 \right] \]  

\[
= \left[ cbx - \frac{cb}{a} x^2 + \frac{cb}{3a^2} x^3 + \frac{ca}{6b^2} \left( b - \frac{b}{a} x \right)^3 \right]_0^a
\]

\[
= \frac{1}{3} c b a.
\]

(10) Reverse the order and evaluate 
\[
\int_0^2 \int_y^2 e^{x^2} dx dy.
\]

**Solution:** Once again,
\[
D = \{(x,y)|0 \leq y \leq 2, y \leq x \leq 2\} = \{(x,y)|0 \leq y \leq 2, 0 \leq y \leq x\}
\]
and
\[
\int_0^2 \int_0^x e^{x^2} dy dx = \int_0^2 x e^{x^2} dx = \frac{1}{2} e^{2} \bigg|_0^2 = \frac{1}{2} (e^4 - 1).
\]

(11) Find the area of the region bounded by \( x = y - y^2 \) and \( y = -x \).

**Solution:** Intersection: \(-y = y - y^2 \Rightarrow y^2 - 2y = y(y - 2) = 0 \Rightarrow y = 0, 2\), then
\[
\int_0^2 \int_{-y}^{y-y^2} dxdy = \int_0^2 \left[ 2y - y^2 \right] dy = y^2 - \frac{1}{3} y^3 \bigg|_0^2 = \frac{4}{3}.
\]