Math 222 Rahman

Homework 2 Due Wednesday, September 28, 2016.

Suggested problems: Sec. 2.1 # 6c, 9c, 17, 19, 22(b,c), 27, 28, 31, 33, 34, 35; Sec. 2.3 # 2, 4, 7, 9, 16, 18a; Sec. 2.7 # 2, 18, 19; Sec. 3.1 # 3, 6, 8, 10, 13, 17, 20, 22.

Mandatory problems:

(1) Consider the IVP, where \( b \) is a constant,
\[
y' = -y + be^{-t}; \quad y(0) = 0.
\]
(a) [5 pts.] Solve the IVP.
(b) [2 pts.] Show that the solution attains its maximum value at \( t = 1 \).
(c) [2 pts.] For what value of \( b \) is this maximum \( y = 2 \)?

(2) Consider the IVP, where \( a \) is a constant,
\[
ty' + (t + 1)y = 2te^{-t}, \quad t > 0; \quad y(1) = a.
\]
(a) [6 pts.] Solve the IVP.
(b) [1 pts.] Show that the solution \( y \to 0 \) as \( t \to \infty \)
(c) [3 pts.] If \( y = 0 \) at \( t = 2 \), what is \( a? \)
(d) [3 pts.] If the solution \( y \) has a critical point at \( t = 1/2 \), what is \( a? \)

(3) Consider two connected tanks: Tank 1 and Tank 2. Initially Tank 1 contains 100 gal of fresh water and Tank 2 100 gal of brine containing 10 lb of salt. Brine containing 0.5 lb/gal of salt is pumped into Tank 1 at 1 gal/min, and the mixture leaves Tank 1 and into Tank 2 and finally out of Tank 2 at the same rate.
(a) [5 pts.] Derive the IVP (i.e. ODE + IC) for the salt content in Tank 1.
(b) [5 pts.] Derive the IVP for the salt content in Tank 2.
(c) [4 pts.] Find the amount of salt in Tank 1 for any time (i.e. solve the IVP).
(d) [6 pts.] Find the amount of salt in Tank 2 for any time.

(4) Consider the IVP \( y' = 2y - 1; \quad y(0) = 1. \)
(a) [5 pts.] Find the exact solution (i.e. solve the IVP) for \( t = 0.2 \).
(b) [10 pts.] Use Euler’s method to approximate the solution for \( h = 0.1, 0.05, 0.01 \).
(c) [3 pts.] Find the error (i.e. difference between approximate solution and exact solution) for each \( h. \)
(d) [1 pts.] Is this error decreasing as \( h \) decreases?

(5) Consider the IVP
\[
2y'' + 3y' - 2y = 0; \quad y(0) = 1, \quad y'(0) = -\beta \quad (\text{with } \beta > 0).
\]
(a) [7 pts.] Solve the IVP.
(b) [7 pts.] Plot the solution for \( \beta = 1. \)
(c) [4 pts.] Find the minimum of the solution.
(d) [2 pts.] Find the smallest (in magnitude) value for \( \beta \) for which the solution has no minimum.

Your homework raw score is: \[ \frac{n}{2m} \cdot M + \left( 1 - \frac{n}{2m} \right) \cdot N = N + \frac{n}{2m} (M - N). \]