Mandatory problems:

(1) [15 pts] Use undetermined coefficients to solve the following IVP
\[ y'' + y = t(1 + \sin t); \quad y(0) = y'(0) = 0. \]

(2) [15 pts] Suppose \( y_1 = x^2 \) and \( y_2 = x^2 \ln x \) are solutions to the following ODE
\[ x^2 y'' - 3xy' + 4y = 0; \quad x > 0 \]
Identify the particular solution and then solve the IVP of
\[ x^2 y'' - 3xy' + 4y = x^2 \ln x; \quad x > 0; \quad y(1) = y'(1) = 0. \]

(3) Consider the following IVP
\[ u'' + \frac{1}{4}u' + 2u = 2 \cos \omega t; \quad u(0) = 0, \quad u'(0) = 2. \]

(a) [13 pts] Determine the steady state solution.

(b) [3 pts] Find the amplitude, A, of the steady state solution in terms of \( \omega \).

(c) [3 pts] Plot \( A \) vs. \( \omega \).

(d) [1 pt] Find the maximum value of \( A \) and the frequency of \( \omega \) for which it occurs.

Your homework raw score is: \( \frac{n}{2m} \cdot M + \left( 1 - \frac{n}{2m} \right) \cdot N = N + \frac{n}{2m} (M - N) \).