Supplementary problems: pg. 64 # 32, 34, 36; pg. 79 # 1, 21, 23, 29

Compulsory problems:

(1) Solve the IVP \( y' = 2y - 1; \ y(0) = 1 \).

(2) Consider the IVP, where \( b \) is a constant,
\[ y' = -y + be^{-t}; \ y(0) = 0. \]

(a) [5 pts.] Solve the IVP.
(b) [2 pts.] Show that the solution attains its maximum value at \( t = 1 \).
(c) [2 pts.] For what value of \( b \) is this maximum \( y = 2 \)?

(3) Consider the IVP, where \( a \) is a constant,
\[ ty' + (t + 1)y = 2te^{-t}, \quad t > 0; \ y(1) = a. \]

(a) [6 pts.] Solve the IVP.
(b) [1 pts.] Show that the solution \( y \to 0 \) as \( t \to \infty \)
(c) [3 pts.] If \( y = 0 \) at \( t = 2 \), what is \( a \)?
(d) [3 pts.] If the solution \( y \) has a critical point at \( t = 1/2 \), what is \( a \)?

(4) Consider two connected tanks: Tank 1 and Tank 2. Initially Tank 1 contains 100 gal of fresh water and Tank 2 100 gal of brine containing 10 lb of salt. Brine containing 0.5 lb/gal of salt is pumped into Tank 1 at 1 gal/min, and the mixture leaves Tank 1 and into Tank 2 and finally out of Tank 2 at the same rate.

(a) [5 pts.] Derive the IVP (i.e. ODE + IC) for the salt content in Tank 1.
(b) [5 pts.] Derive the IVP for the salt content in Tank 2.
(c) [4 pts.] Find the amount of salt in Tank 1 for any time (i.e. solve the IVP).
(d) [6 pts.] Find the amount of salt in Tank 2 for any time.

A word on how the grading will work: Let \( m \) be the number of supplementary problems, \( n \) the number of supplementary problems completed, \( M \) the total number of points for the compulsory problems, and \( N \) the number of points earned for the compulsory problems. Then your homework score is:
\[ \frac{n}{2m} \cdot M + \left( 1 - \frac{n}{2m} \right) \cdot N = N + \frac{n}{2m} (M - N). \] Just be glad it’s not a differential equation.