Supplementary problems: # 1, 3, 5, 7

Compulsory problems:

(1) Consider the IVP

\[ 2y'' + 3y' - 2y = 0; \ y(0) = 1, \ y'(0) = -\beta \text{ (with } \beta > 0). \]

(a) [7 pts.] Solve the IVP.
(b) [7 pts.] Plot the solution for \( \beta = 1 \).
(c) [4 pts.] Find the minimum of the solution.
(d) [2 pts.] Find the smallest (in magnitude) value for \( \beta \) for which the solution has no minimum.

(2) Consider the IVP \( 9y'' + 12y' + 4y = 0; \ y(0) = a > 0, \ y'(0) = -1 \).

(a) [7 pts] Solve the IVP.
(b) [3 pts] Find the critical value of \( a \) that separates solutions that become negative from those that are always positive.

(3) Consider the IVP: \( y'' + 2y' + 6y = 0; \ y(0) = 2, \ y'(0) = \alpha \geq 0 \).

(a) [6 pts] Solve the IVP.
(b) [4 pts] Find \( \alpha \) such that \( y = 0 \) when \( t = 1 \).

Your homework raw score is: \( \frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m} (M - N) \).