Supplementary problems: Sec. 3.4 # 18; 3.5 # 5, 9; 3.7 # 3, 7

Compulsory problems:
Since these sequences are nontrivial, please use induction whenever proving increasing/decreasing.

1) [15 pts.] Prove the sequence \( \{a_n\} \) converges if \( |a_n| < 2 \) and \( |a_{n+2} - a_{n+1}| \leq \frac{1}{8} |a_{n+1}^2 - a_n^2| \).
   (Hint: Can you write \( |a_m - a_n| \) in terms of \( |a_2 - a_1|? \)
   (Do not use the fact that this is a contractive sequence.)

2) [15 pts.] Prove that # 5 in the book \( x_n = \sqrt{n} \) is not a contractive sequence. (Hint: Use contradiction.)
   (If you compute any limits don’t use the definition, just use Calc II techniques.)

3) [5 pts.] Prove (by using partial sums; i.e. different from the proof in the book) that the following series diverges:
   \[
   \sum_{n=1}^{\infty} \frac{1}{n}
   \]  
(1)

4) [5 pts.] Give an example of a sequence where \( |x_{n+1} - x_n| \to 0 \), but for all \( N \in \mathbb{N} \) there is an \( \varepsilon > 0 \) such that there is some \( n, m \geq N \Rightarrow |x_m - x_n| \geq \varepsilon \). Show why your example works.

Your homework raw score is: \( \frac{n}{2m} \cdot M + \left( 1 - \frac{n}{2m} \right) \cdot N = N + \frac{n}{2m} (M - N) \). For this homework, \( M = 40 \), \( m = 5 \), \( N \) is the number of compulsory problems you get correct, and \( n \) is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for full completion, but I won’t take off points for mistakes.